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Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Tuesday 13 October 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WMA14/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

Make an Assumption:

"Assume there exists a number m such that when m^3 is even, m is odd" B1

we want our assumption to be the opposite of what we are trying to prove.

In the Proof, we are trying to contradict the assumption

If m is odd, we can write it as $m = 2k+1$ where k is an integer. M1

Hence:

$$m^3 = (2k+1)^3$$

$$= (2k)^3 + (2k)^2(1) + (2k)(1)^2 + 1^3 \quad \text{expand}$$

$$= 8k^3 + 4k^2 + 2k + 1 \quad \text{simplify}$$

$$= 2(4k^3 + 2k^2 + k) + 1 \quad \text{A1}$$

$2(4k^3+2k^2+k)+1$ is odd, hence any number $\times 2$ is even, so any number $\times 2 + 1$ is odd. This is a contradiction, so if m^3 is even, n is even too. A1

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Question 1 continued

Q1

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2. (a) Use the binomial expansion to expand

$$(4-5x)^{-\frac{1}{2}} \qquad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2 giving each coefficient as a fully simplified fraction.

(4)

$$f(x) = \frac{2+kx}{\sqrt{4-5x}} \qquad \text{where } k \text{ is a constant and } |x| < \frac{4}{5}$$

Given that the series expansion of f(x), in ascending powers of x, is

$$1 + \frac{3}{10}x + mx^2 + \dots \qquad \text{where } m \text{ is a constant}$$

(b) find the value of k,

(2)

(c) find the value of m.

(2)

(a) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$, Range of validity $|x|<1$

We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$(4-5x)^{-\frac{1}{2}} = (4)^{-\frac{1}{2}}\left(1-\frac{5}{4}x\right)^{-\frac{1}{2}}$$
$$= \frac{1}{2}\left(1-\frac{5}{4}x\right)^{-\frac{1}{2}} \quad \text{B1}$$

Substitute into the Formula:

$$\frac{1}{2}\left(1-\frac{5}{4}x\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(-\frac{5}{4}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{5}{4}x\right)^2 + \dots\right) \quad \text{M1A1}$$

$$= \frac{1}{2}\left(1 + \frac{5}{8}x + \frac{\frac{3}{4}}{2}\left(\frac{25}{16}x^2\right) + \dots\right)$$

$$= \frac{1}{2}\left(1 + \frac{5}{8}x + \frac{75}{128}x^2 + \dots\right)$$

$$= \frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2 + \dots \quad \text{A1}$$

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Question 2 continued

(b) $f(x) = (4-5x)^{-\frac{1}{2}}(2+kx)$

we got this expansion in (a)

$\Rightarrow \left(\frac{1}{2} + \frac{5}{16}x + \frac{75}{256}x^2\right)(2+kx)$

Use the coefficient of x to get the value of k:

given $\frac{3}{10}x = \frac{10}{16}x + \frac{1}{2}kx$

$\frac{3}{10} = \frac{10}{16} + \frac{1}{2}k$

$\frac{1}{2}k = -\frac{13}{40} \Rightarrow k = -\frac{13}{20}$ value of k

M1A1

(c) We want the coefficient of x^2:

got in (b)

$m\cancel{x^2} = \frac{5}{16}\left(-\frac{13}{20}\right)\cancel{x^2} + \frac{150}{256}\cancel{x^2}$

$m = \frac{5}{16}\left(-\frac{13}{20}\right) + \frac{75}{128}$

$m = \frac{49}{128}$ value of m

M1A1

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Question 2 continued

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Question 2 continued

Q2

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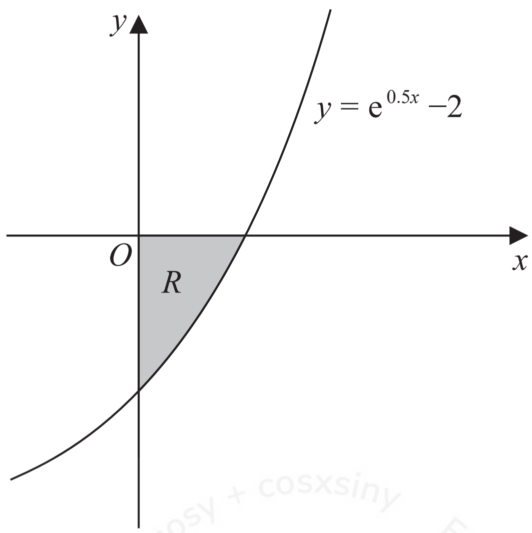

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$

The region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The region R is rotated 360° about the x-axis to form a solid of revolution.

Show that the volume of this solid can be written in the form $a \ln 2 + b$, where a and b are constants to be found.

(6)

Formula for Volume of Revolution around the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

where a and b are the points on the x-axis bounding the area

Let's get the limits:

at $y=0 \Rightarrow$ upper bound

$$0 = e^{\frac{1}{2}x} - 2$$

$$2 = e^{\frac{1}{2}x}$$

$$\ln 2 = \ln e^{\frac{1}{2}x} \quad \text{apply ln's on both sides}$$

$$\ln 2 = \frac{1}{2}x \quad \text{apply } \ln e^x = x$$

$$x = 2\ln 2$$

lower bound : $x = 0$ B1

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Question 3 continued

Hence:

$$V = \pi \int_0^{2\ln 2} (e^{\frac{1}{2}x} - 2)^2 \, dx$$

$$= \pi \int_0^{2\ln 2} e^x - 4e^{\frac{1}{2}x} + 4 \, dx \qquad \text{expand bracket}$$

$$= \pi \left[e^x - \frac{4}{\frac{1}{2}} e^{\frac{1}{2}x} + 4x \right]_0^{2\ln 2}$$

$$= \pi \left[e^x - 8e^{\frac{1}{2}x} + 4x \right]_0^{2\ln 2} \qquad \text{M1A1A1}$$

$$= \pi\left((e^{2\ln 2} - 8e^{\ln 2} + 4(2\ln 2)) - (e^0 - 8e^0 + 4(0)) \right)$$

$$= \pi(e^{\ln 4} - 8e^{\ln 2} + 8\ln 2 - 1 + 8) \qquad \text{use ln law } a\ln b = \ln b^a$$

$$= \pi(4 - 8 \times 2 + 8\ln 2 + 7) \qquad \text{use ln law } \ln e^x = x$$

$$= \pi(8\ln 2 - 5) \qquad \text{dM1}$$

$$\therefore \text{Volume} = 8\pi\ln 2 - 5\pi \qquad \text{A1}$$

Q3

(Total 6 marks)

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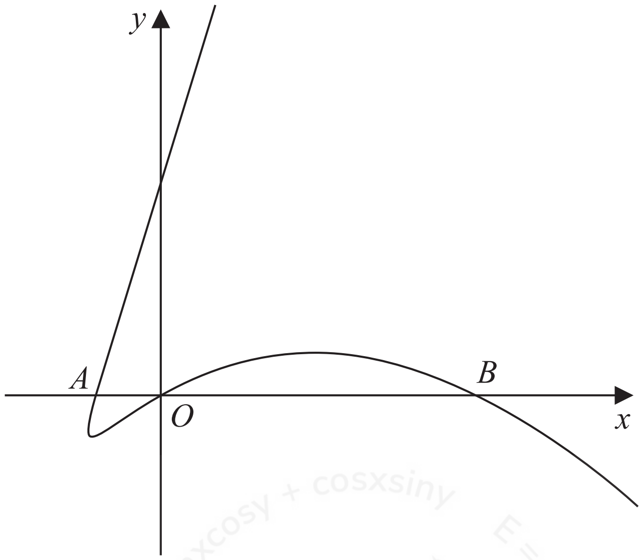

Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

(a) Coordinates of A and B are on the x-axis, $\therefore$ $y=0$.

$0 = t^3 - 4t$

$0 = t(t^2 - 4)$

$t=0$, $t = \pm 2$ M1

Substitute $t=\pm 2$ into $x = \ldots$

$x_A = 2(2)^2 - 6(2)$ $\qquad$ $x_B = 2(-2)^2 - 6(-2)$

$= 8 - 12$ $\qquad$ $= 8 + 12$

$x_A = -4$ $\qquad$ $x_B = 20$

$\therefore$ $A(-4, 0)$ A1 $\qquad$ $\therefore$ $B(20, 0)$ B1

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Question 4 continued

(b) We need to get $\frac{dy}{dx}$ to get the gradient of the tangent.

★ Parametric differentiation:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$ get there by differentiating the parametrics separately.

$y = t^3 - 4t$

$\frac{dy}{dt} = 3t^2 - 4$

$x = 2t^2 - 6t$

$\frac{dx}{dt} = 4t - 6$

$$\frac{dy}{dx} = \frac{3t^2 - 4}{4t - 6}$$ M1A1

We need $\frac{dy}{dx}$ at $t = -2$ (Point B).

$$\left.\frac{dy}{dx}\right|_{t=-2} = \frac{3(-2)^2 - 4}{4(-2) - 6} = -\frac{4}{7} \quad m$$

M1

Use formula $y - y_1 = m(x - x_1)$ to get the equation of the line:

$m = -\frac{4}{7}$, $B(20, 0)$:

$$y - 0 = -\frac{4}{7}(x - 20) \quad \text{M1}$$

$$7y = -4x + 80$$

$$4x + 7y - 80 = 0$$ hence shown

A1

(c) Substitute the $x = \ldots$ and $y = \ldots$ into our tangent equation to get t at P.

$$4(2t^2 - 6t) + 7(t^3 - 4t) - 80 = 0 \quad \text{M1}$$

$$8t^2 - 24t + 7t^3 - 28t - 80 = 0$$

$$7t^3 + 8t^2 - 52t - 80 = 0 \quad \text{A1}$$

factorizing is easy since we know $t = -2$ is a solution

$$(t+2)^2(7t - 20) = 0$$

$t = \frac{20}{7}$ t at P.

Substitute $t = \frac{20}{7}$ into $x = \ldots$:

$$x = 2\left(\frac{20}{7}\right)^2 - 6\left(\frac{20}{7}\right) \quad \text{dM1}$$

$$\Rightarrow x = -\frac{40}{49}$$ x-coordinate of P

A1

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Question 4 continued

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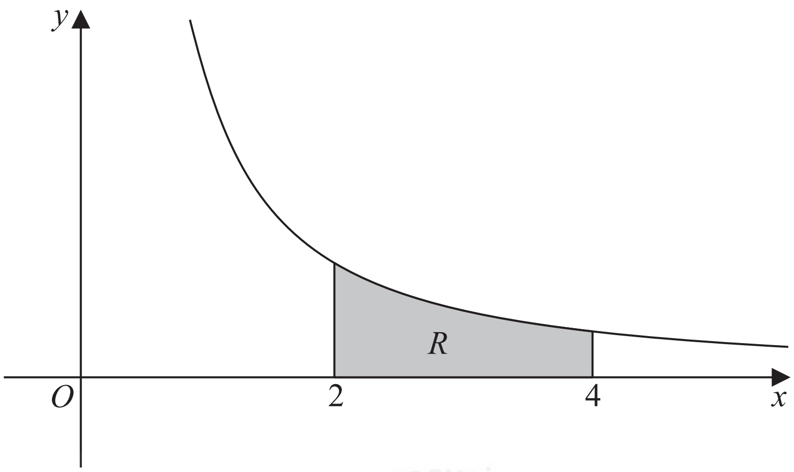

Figure 3

(a) Find $\displaystyle\int \frac{\ln x}{x^2}\,dx$ **(3)**

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of R, writing your answer in simplest form. **(4)**

(a) $\displaystyle\int x^{-2}\ln x \; dx$

multiplication ∴ By Parts
$\int uv'\,dx = uv - \int u'v\,dx$

$u = \ln x \xrightarrow{\frac{d}{dx}} u' = \frac{1}{x}$

$v = -x^{-1} \xleftarrow{\int} v' = x^{-2}$

$= -\frac{1}{x}\ln x - \int -\frac{1}{x} \times \frac{1}{x}\,dx$

M1

$= -\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$

$= -\frac{\ln x}{x} - \frac{1}{x} + c$ hence found

dM1A1

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Question 5 continued

(a) We are asked to find:

$$\int_2^4 \frac{3+2x+\ln x}{x^2}\,dx$$

$$= \int_2^4 \frac{3}{x^2} + \frac{2x}{x^2} + \frac{\ln x}{x^2}\,dx$$ split the fraction

we found this in (a)

$$= \int_2^4 \frac{3}{x^2} + \frac{2}{x} + \frac{\ln x}{x^2}\,dx$$

B1

$$= \left[-\frac{3}{x} + 2\ln x - \frac{\ln x}{x} - \frac{1}{x}\right]_2^4$$ use our result from (a)

M1

$$= \left(\left(-\frac{3}{4} + 2\ln 4 - \frac{\ln 4}{4} - \frac{1}{4}\right) - \left(-\frac{3}{2} + 2\ln 2 - \frac{\ln 2}{2} - \frac{1}{2}\right)\right)$$

M1

$$= -1 + 2\ln 2^2 - \frac{1}{4}\ln 2^2 + \frac{3}{2} - 2\ln 2 + \frac{1}{2}\ln 2 + \frac{1}{2}$$ convert $4 = 2^2$

$$= 1 + 4\ln 2 - \frac{1}{2}\ln 2 - 2\ln 2 + \frac{1}{2}\ln 2$$ apply ln law $\ln a^b = b\ln a$

area $= 1 + 2\ln 2$ A1 collect like terms

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Question 5 continued

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Question 5 continued

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6. A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x\ln x = 0$$

(2)

(a)

$$y = x^{\sin x}$$

$$\ln y = \ln x^{\sin x}$$ apply ln's

$$\ln y = \sin x \ln x$$ apply ln law: $\ln a^b = b\ln a$

★ Implicit differentiation: B1

When differentiating y, multiply by $\frac{dy}{dx}$

Product Rule: $\frac{d}{dx}(uv) = uv' + u'v$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\sin x + \cos x \ln x$$ Differentiate

M1M1A1

$$\frac{dy}{dx} = \frac{y\sin x}{x} + y\cos x \ln x$$ isolate $\frac{dy}{dx}$ on LHS

A1

(b) At stationary points, $\frac{dy}{dx} = 0$.

$$\therefore\ 0 = \frac{y\sin x}{x} + y\cos x\ln x$$ divide both sides by y

$$0 = \frac{\sin x}{x} + \cos x \ln x$$ M1

$$0 = \frac{x \cdot \sin x}{x\cos x} + \frac{x \cdot \cos x \ln x}{\cos x}$$ multiply all by x and divide all by $\cos x$

$$0 = \frac{\sin x}{\cos x} + x\ln x$$

$$0 = \tan x + x\ln x$$ use $\frac{\sin x}{\cos x} = \tan x$

A1 hence shown

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Question 6 continued

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7. (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx$$

(Solutions relying entirely on calculator technology are not acceptable.)

(6)

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,dx$$

(6)

(i) Use the substitution $u=\sqrt{2x-1}$. We need new limits and $dx=\ldots$ to integrate.

New limits:

x		u
5	$\sqrt{2(5)-1} \rightarrow$	3
1	$\sqrt{2(1)-1} \rightarrow$	1

Differentiate to get $dx=\ldots$

$u=(2x-1)^{\frac{1}{2}}$

$\frac{du}{dx}=\frac{1}{2}(2x-1)^{-\frac{1}{2}}\times 2$

$=(2x-1)^{-\frac{1}{2}}$

$dx=(2x-1)^{\frac{1}{2}}\,du$ B1

Substitute into the Integral:

new limits

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx$$

u

$$=\int_1^3 \frac{3x}{\cancel{\sqrt{2x-1}}}\times\cancel{\sqrt{2x-1}}\,du$$

$$=\int_1^3 3x\,du$$

$u=\sqrt{2x-1} \Rightarrow \frac{u^2+1}{2}=x$

$$=\int_1^3 \frac{3(u^2+1)}{2}\,du$$

$$=\frac{3}{2}\int_1^3 u^2+1\,du \quad \text{M1}$$

dM1A1 $$=\frac{3}{2}\left[\frac{1}{3}u^3+u\right]_1^3$$ Integrate

$$=\frac{3}{2}\left(\left(\tfrac{1}{3}(3)^3+3\right)-\left(\tfrac{1}{3}(1)^3+1\right)\right) \quad \text{M1}$$

$$=\frac{3}{2}\left(12-\frac{4}{3}\right)$$

$$=16 \quad \text{A1}$$

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Question 7 continued

(ii) We need to do Partial Fractions

$$\frac{6x^2-16}{(x+1)(2x-3)} = A + \underbrace{\frac{B}{(x+1)} + \frac{C}{(2x-3)}}$$

Since the numerator has a squared term, we will have a quotient (A)

Manipulate to get common denominator

Equate the numerators and substitute in values in order to get A, B and C.

$6x^2 - 16 = A(x+1)(2x-3) + B(2x-3) + C(x+1)$

choose values to substitute in so that brackets become 0, to make your life easier!

$x = -1 \Rightarrow 6 - 16 = \cancel{A(0)(2(-1)-3)}^{0} + B(-2-3) + \cancel{C(0)}^{0}$

$-10 = -5B \Rightarrow B = 2$

$x = \frac{3}{2} \Rightarrow \frac{27}{2} - 16 = \cancel{A(\frac{3}{2}+1)(0)}^{0} + \cancel{B(0)}^{0} + C(\frac{3}{2}+1)$

$-\frac{5}{2} = \frac{5}{2}C \Rightarrow C = -1$

$x = 1 \Rightarrow 6(1)^2 - 16 = A(1+1)(2(1)-3) + 2(2(1)-3) - (1+1)$

$-10 = A(2)(-1) - 2 - 2$

$-6 = -2A \Rightarrow A = 3$

$$\therefore \frac{6x^2-16}{(x+1)(2x-3)} = 3 + \frac{2}{(x+1)} - \frac{1}{(2x-3)}$$ B1M1A1

Hence:

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,dx = \int 3 + \frac{2}{(x+1)} - \frac{1}{(2x-3)}\,dx$$ M1

$$= 3x + 2\ln|x+1| - \frac{\ln|2x-3|}{2}$$

Reverse chain Rule: divide by derivative of the bracket

$$= 3x + 2\ln|x+1| - \frac{1}{2}\ln|2x-3| + c$$ A1A1

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Question 7 continued

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Question 7 continued

Q7

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8. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X,

(a) find the position vector of X.

(5)

The point $P(10, -7, 0)$ lies on l_1

The point Q lies on l_2

Given that $\overrightarrow{PQ}$ is perpendicular to l_2

(b) calculate the coordinates of Q.

(5)

(a) Equate the general points of the lines.

$$\begin{pmatrix} 4+3\lambda \\ -3-2\lambda \\ 2-\lambda \end{pmatrix} = \begin{pmatrix} 2+2\mu \\ -\mu \\ -9-3\mu \end{pmatrix}$$

Use x and y components to solve simultaneously for μ and λ:

$$4+3\lambda = 2+2\mu$$

$$-3-2\lambda = -\mu \quad (\times 2) \quad -6-4\lambda = -2\mu \quad +$$

use elimination method

$$-2-\lambda = 2$$

M1

$$\lambda = -4 \Rightarrow -6-4(-4) = -2\mu$$

$$\mu = -5$$

M1A1

Substitute into z component to check that they intersect:

$$2-(-4) = -9-3(-5)$$

$$6 = -9+15$$

$$6 = 6 \checkmark$$

Hence point of intersection using l_1:

$$\mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} - 4\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$$

dM1

$$= \begin{pmatrix} -8 \\ 5 \\ 6 \end{pmatrix}$$ point of intersection

A1

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Question 8 continued

(b) Since Q lies on l_2, we can take Q as the general point of l_2.

$$\vec{OQ} = \begin{pmatrix} 2+2\mu \\ -\mu \\ -9-3\mu \end{pmatrix}$$

Get $\vec{PQ}$:

$$\vec{PQ} = \vec{OQ} - \vec{OP} \text{ given}$$

$$= \begin{pmatrix} 2+2\mu \\ -\mu \\ -9-3\mu \end{pmatrix} - \begin{pmatrix} 10 \\ -7 \\ 0 \end{pmatrix}$$

$$\vec{PQ} = \begin{pmatrix} -8-2\mu \\ 7-\mu \\ -9-3\mu \end{pmatrix} \quad \text{M1}$$

We're given that $\vec{PQ}$ is perpendicular to l_2 $\therefore$ $\vec{PQ} \cdot$ direction vector $= 0$

Formula for dot product:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$$

$$\begin{pmatrix} -8-2\mu \\ 7-\mu \\ -9-3\mu \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} = 0 \quad \text{dM1}$$

$$2(-8-2\mu) - 1(7-\mu) - 3(-9-3\mu) = 0 \quad \text{Solve for } \mu$$

$$-16 + 4\mu - 7 + \mu + 27 + 9\mu = 0$$

$$14\mu = -4$$

$$\text{A1} \quad \mu = -\frac{2}{7} \quad \text{value of } \mu$$

Substitute our found μ value into the general point of l_2 to get the coordinates of Q.

$$\vec{OQ} = \begin{pmatrix} 2+2\mu \\ -\mu \\ -9-3\mu \end{pmatrix}$$

$$\vec{OQ} = \begin{pmatrix} 2+2\left(-\frac{2}{7}\right) \\ -\left(-\frac{2}{7}\right) \\ -9-3\left(-\frac{2}{7}\right) \end{pmatrix} \quad \text{ddM1}$$

$$\vec{OQ} = \begin{pmatrix} \frac{10}{7} \\ \frac{2}{7} \\ -\frac{57}{7} \end{pmatrix} \Rightarrow Q\left(\frac{10}{7}, \frac{2}{7}, -\frac{57}{7}\right) \quad \text{A1}$$

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Question 8 continued

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Question 8 continued

Q8

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9. Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, A cm^2, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r}\right)^2$$

where p, q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

(2)

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Question 9 continued

(a) We need to solve the Differential Equation.

$$\frac{dA}{dt} = \frac{A^{\frac{3}{2}}}{5t^2}$$

$$\int \frac{1}{A^{\frac{3}{2}}}\, dA = \int \frac{1}{5t^2}\, dt$$ Separate Variables by grouping the like terms on one side M1

$$\int A^{-\frac{3}{2}}\, dA = \frac{1}{5}\int t^{-2}\, dt$$

$$\frac{1}{-\frac{1}{2}} A^{-\frac{1}{2}} = \frac{1}{5} \times \frac{1}{-1} \times t^{-1} + c$$

$$\frac{-2}{\sqrt{A}} = -\frac{1}{5t} + c$$ M1M1A1

Substitute in A = 2.25, t = 3 and solve for c:

$$\frac{-2}{\sqrt{2.25}} = -\frac{1}{5(3)} + c$$

$$\frac{1}{15} - \frac{2}{\frac{3}{2}} = c \Rightarrow c = -\frac{19}{15}$$ M1

Substitute c back in and manipulate to get the needed form:

$$\frac{-2}{\sqrt{A}} = -\frac{1}{5t} - \frac{19}{15}$$

$$\frac{2}{\sqrt{A}} = \frac{1}{5t} + \frac{19}{15}$$ multiply both sides by -1

$$\frac{2}{\sqrt{A}} = \frac{3 + 19t}{15t}$$ do common denominator

$$\frac{1}{\sqrt{A}} = \frac{3 + 19t}{30t}$$ divide both sides by 2

$$\left(\frac{1}{\sqrt{A}}\right)^{-1} = \left(\frac{3 + 19t}{30t}\right)^{-1}$$

$$\sqrt{A} = \frac{30t}{19t + 3}$$

$$A = \left(\frac{30t}{19t + 3}\right)^2$$ square both sides

M1A1

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Question 9 continued

(b) $A = \left(\dfrac{\frac{30t}{t}}{\frac{19t}{t} + \frac{3}{t}} \right)^2$ divide all of RHS by t

As $t \to \infty$: $A = \left(\dfrac{30}{19 + \frac{3}{t}} \right)^2$ ↳ as $t \to \infty$, $\frac{3}{t} \to 0$

$\Rightarrow A = \left(\dfrac{30}{19} \right)^2$ M1

$\therefore$ As $t \to \infty$, $A = \dfrac{900}{361} = 2.49\text{cm}^2$ A1

Q9

(Total 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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